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# Dynamic balancing scheme for motor armatures

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**Abstract**

This paper presents a novel approach based on an adaptive parameter estimation technique for the dynamic balancing of the armature in an automobile starting motor. The proposed scheme is implemented on an experimental system comprising an unbalance measurement machine and a milling machine. Applying the influence coefficient method, two matrices, namely the unbalance calculation matrix and the milling vector calculation matrix, are constructed. The unbalance calculation matrix computes the armature unbalance based on vibration measurements, while the milling vector calculation matrix establishes the position and magnitude of the required unbalance correction. To compensate for the wear of the milling cutter following repeated machining operations, an algorithm is formulated to generate on-line estimates of the parameter settings required in the milling vector calculation matrix. A series of balancing experiments are performed to evaluate the feasibility and performance of the proposed scheme. The results show that the balancing system achieves a better balancing performance than a system based on the conventional influence coefficient method.


## 1. Introduction

One of the major sources of vibration in an electric motor is unbalance in the armature. Vibration not only results in a noisy motor operation, but also severely limits the service life of the motor bearings. Accordingly, in the motor assembly line, each armature must be properly balanced before it is assembled to ensure that the motor vibration will fall within acceptable limits.

Balancing techniques are commonly classified as either static (i.e. one-plane balancing) or dynamic (i.e. two-plane balancing) [1]. Static balancing is applied primarily to rotors with a length/diameter ($L/D$) ratio of less than 1. One-plane balancing is generally performed using a four-run approach and the single-plane influence coefficient method. Dynamic balancing is generally applied for the balancing of rotors with a $L/D$ ratio of more than 1, e.g. the armatures in generators or motors. Typical dynamic balancing techniques include the modal balancing method, the influence coefficient method, and the unified balancing method [2]. The modal balancing method is generally applied for the balancing of flexible shafts. However, obtaining accurate results requires a knowledge of the rotor's modal shapes. By contrast, the influence coefficient method is typically employed for the balancing of rigid shafts or rotors. However, this method requires highly accurate

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**Nomenclature**

| | |
|---|---|
| $Y$ | vector of vibration measurement |
| $X$ | milling vector |
| $\angle$ | phase angle of a complex number |
| $U$ | unbalance vector |
| $U_c$ | correction mass vector |
| $U_r$ | residual unbalance vector |
| $\theta_{YX}$ | influence coefficient matrix mapping from $Y$ to $X$ |
| $\hat{\theta}$ | estimation of $\theta$ |

measurements of the vibration in order to guarantee an acceptable balancing result. The unified balancing method aims to exploit the particular benefits of both methods, while simultaneously eliminating their limitations.

The influence coefficient method has become a mainstream method for the dynamic balancing of rigid body rotors [2]. Thearle [3] expanded the basic principles of this method to develop a two-plane balancing technique for the dynamic balancing of rigid motors using a two-plane, two-sensor, single-speed, and exact-point influence coefficient balancing procedure. Various researchers have proposed improvements to the original influence coefficient balancing method. For example, Goodman [4] extended the basic influence coefficient procedure by including the least-squares (LS) method to improve its accuracy. Lund and Tonneson [5] developed a procedure to optimize the influence coefficient matrix by taking the effects of vibration measurement errors into account. While both approaches provided excellent results for their own particular experimental systems, most studies of the influence coefficient method reported in the literature focus primarily on field balancing. By contrast, the task of balancing rigid rotors on a balancing machine in the motor assembly line has attracted comparatively little attention.

In the motor manufacturing industry, rotor unbalance is generally corrected using some form of material removal technique. Such techniques are suitable for high production rate environments and provide acceptable results when lower balancing tolerances are sufficient [6]. In material removal methods, the unbalance amount is converted to a milling vector which specifies both the depth and the position of the milling slots which are to be machined in the center of the poles on the laminations of the motor armature to correct the unbalance. However, the unbalance amount of motor armatures is generally quite large, and hence many cuts are required to compensate for the unbalance. To reduce the cost of the balancing process, it is therefore necessary to optimize the relationship between the unbalance amount and the milling vector. However, the relationship between the cutting depth of the milling machine and the weight of the removed material depends on the wear condition of the cutter. As a result, the optimal parameters settings specified in the matrix vary over time. Therefore, a requirement exists for a dynamic balancing system in which these parameters can be tuned on-line to compensate for the progressive wear of the milling machine.

Accordingly, this paper develops a dynamic balancing system comprising an unbalance measurement machine and a milling machine for the balancing of automobile starting motor armatures. The paper commences by outlining the basic principles of the proposed balancing method and then describes the enhancement of this balancing method via its integration with an adaptive parameter estimation scheme. Finally, a series of experiments are performed to verify the proposed approach and to evaluate its performance compared to that of a balancing scheme based on the conventional influence coefficient method.

## 2. Design of dynamic balancing system

In balancing a rotor, the most important consideration is the balancing quality required. This requirement is conventionally expressed in terms of the acceptable balance tolerance. The International Standards Organization (ISO) has developed the ISO 1940 Standard for determining the balance tolerance (defined in terms of the permissible residual unbalance) for various rotor classifications [7]. According to this standard, rotors are assigned a "G" grade ranging from G0.4 to G4000 in accordance with their intended application. In general, motor armatures such as those considered in the present study are generally assigned grades of G1, G2.5 or G6.3. ISO 1940 is widely applied throughout the motor industry and is therefore used in the present study to evaluate the performance of the proposed dynamic balancing system.

The dynamic rotor balancing system proposed in this study is based on the influence coefficient method [8,9]. In conventional balancing methods, the influence coefficient matrix is used to relate the amount of unbalance to the measured vibration of the rotor. However, in the method proposed in this study, the influence coefficient matrix is employed to relate the material removal milling vector to the vibration of the armature. Furthermore, the designed balancing system incorporates an adaptive parameter identification technique which allows the matrix parameters to be updated on-line during the balancing process in order to compensate for errors arising during transformation between the vibration measurements and the milling vector and to adapt to the changing wear condition of the milling machine. As described in the paragraphs below, the balancing scheme basically comprises an unbalance calculation module, an unbalance correction module, and a parameter estimation module.

### 2.1. Calculation of amplitude and phase of vibration signals

When an unbalanced rotor rotates on a soft-suspended support, the amplitude of the associated vibration is proportional to the amount of unbalance. As a result, the unbalance amount can be determined by calculating the vibrational amplitude. In the proposed balancing system, the signals from the vibration sensors are filtered and amplified and the amplitude and phase angle of the vibration measurement, $Y(t)$, are then determined using the Fourier series.

In general, any periodic signal can be expanded by the Fourier series as

$$\begin{aligned} Y(t) &= a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t) \\ &= a_0 + a_1 \cos \omega t + b_1 \sin \omega t + a_2 \cos 2\omega t + b_2 \sin 2\omega t + \dots, \end{aligned} \tag{1}$$

where

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} Y(t) \cos n\omega t \, dt, \quad \text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} Y(t) \sin n\omega t \, dt. \tag{2}$$

In the current application, the fundamental frequency $\omega$ in Eqs. (1) and (2) corresponds to the balancing speed of the rotor. Since the vibration signals are dominated by the fundamental component, Eq. (1) can be approximated by

$$Y(t) \approx a_0 + a_1 \cos \omega t + b_1 \sin \omega t. \tag{3}$$

Furthermore, since the vibration sensors used in the proposed balancing system are of the velocity type, $a_0 = 0$, and therefore it can be shown that $Y(t)$ is given by

$$Y(t) = y \cos(\omega t - \phi_1) = y\angle\phi_1, \tag{4}$$

with $y = \sqrt{a_1^2 + b_1^2}$, $\phi_1 = \tan^{-1}(b_1/a_1)$, and $a_1 = (1/\pi)\int_{-\pi}^{\pi} Y(t) \cos \omega t \, dt$, $b_1 = (1/\pi)\int_{-\pi}^{\pi} Y(t) \sin \omega t \, dt$.

During balancing, the rotor speed, $\omega$, is known a priori, and hence the amplitude and phase angle of $Y(t)$ can be determined from Eq. (4).

### 2.2. Determination of amount of unbalance

In the proposed balancing system, the rotor unbalance is computed using the influence coefficient method. In applying this method, the following assumptions are made: (1) the vibratory response of the unbalanced rotor is proportional to the amount of the unbalance, and (2) the unbalance amount is negligible compared to the total weight of the rotor. Fig. 1 presents the basic configuration of the motor armature considered in the present study. Importantly, the armature can be regarded as a rigid body because its service speed when assembled in a motor is far lower than its natural frequency. Furthermore, the $L/D$ ratio of the armature is greater than 1, and hence dynamic balancing rather than static balancing is required. In the influence

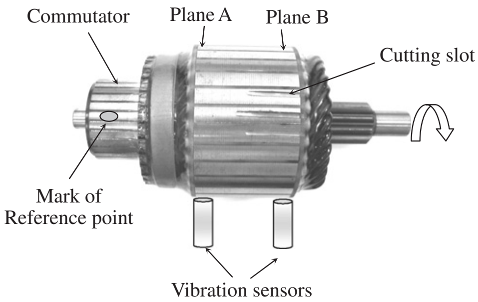


Fig. 1. Schematic diagram of armature showing two balance planes.

coefficient method, all of the unbalances can be modeled as though they occur on two distinct planes on the armature. Conventionally, these planes are referred to as the balancing planes. During the balancing operation, vibration readings are acquired from these balancing planes using two vibration sensors, and a reference point is used to identify the phase angle of the unbalance.

The vibration readings $Y_A$ and $Y_B$ obtained on planes $A$ and $B$, respectively, (see Fig. 1) are assumed to be linear combinations of the unknown unbalances $U_A$ and $U_B$ on planes $A$ and $B$, respectively, i.e.

$$\begin{bmatrix} Y_A \\ Y_B \end{bmatrix} = \begin{bmatrix} \theta_{AA} & \theta_{AB} \\ \theta_{BA} & \theta_{BB} \end{bmatrix} \begin{bmatrix} U_A \\ U_B \end{bmatrix} = \theta U, \tag{5}$$

where $\theta$ is the influence coefficient matrix and its entries are all complex numbers. In matrix $\theta$, each entry, $\theta_{ij}$, is known as an influence coefficient and describes the effect of an unbalance in the $j$th plane on the vibration in the $i$th plane. To determine these influence coefficients, a trial mass $M_j$ with a mass of approximately five times the permissible residual unbalance is positioned in one plane and the corresponding vibratory response to this mass is measured on the measuring plane. The procedure is then repeated for the other plane. The influence coefficient is then computed from the measurement results in accordance with

$$\theta_{ij} = \frac{Y_i^j - Y_j}{M_j}, \quad \text{with } i,j = A, B, \tag{6}$$

where $Y_j$ is the $j$th vibration reading without the trial mass installed, $Y_i{}^j$ is the $i$th vibration reading with the trial mass installed on the $j$th balancing plane, and $M_j$ is a complex number representing the amplitude and angular position of the trial mass. Having constructed the influence coefficient matrix, the rotor unbalances can then be determined from

$$U = \begin{bmatrix} U_A \\ U_B \end{bmatrix} = \begin{bmatrix} \theta_{AA} & \theta_{AB} \\ \theta_{BA} & \theta_{BB} \end{bmatrix}^{-1} \begin{bmatrix} Y_A \\ Y_B \end{bmatrix}. \tag{7}$$

It follows that the correction masses which should be added on planes $A$ and $B$ are given by

$$U_c = -U = [U_{cA} \ \ U_{cB}]^{\mathrm{T}}, \tag{8a}$$

with

$$U_{cA} = \frac{\theta_{BB} Y_A - \theta_{AB} Y_B}{\theta_{AA}\theta_{BB} - \theta_{AB}\theta_{BA}} \quad \text{and} \quad U_{cB} = \frac{\theta_{AA} Y_B - \theta_{BA} Y_A}{\theta_{AA}\theta_{BB} - \theta_{AB}\theta_{BA}}. \tag{8b}$$

When balancing a rotor using the mass addition technique, Eq. (8) allows appropriate values for the correction masses $U_{cA}$ and $U_{cB}$ to be computed once the influence coefficient matrix has been determined. As shown in Eq. (7), the influence coefficient matrix relates the unbalance to the vibratory response. Although this matrix provides a convenient solution when balancing rotors using a mass addition technique, in mass removal methods, an additional relationship is required to map the weight of the material removed from the armature to the cutting depth of the milling cutter. In a practical milling machine, this relationship is nonlinear and varies as a function of the wear condition of the cutter. Since it may be necessary to balance thousands of motor armatures in a single day in a motor mass-production line, the effects of cutter wear must be taken into account. Therefore, the dynamic balancing scheme proposed in this study incorporates an adaptive parameter estimation algorithm which enables the parameters in the influence coefficient matrix to be tuned on-line during the balancing process.

### 2.3. Adaptive parameter estimation

The least-squares (LS) method is widely applied to solve linear optimization problems. The goal of the LS method is to identify the model output, $\hat{y}$, which best approximates the process output, $y$, in the LS sense, i.e. with the minimal sum of the squared error loss function values. This process is trivial for a mathematical model which can be written in the form of

$$\hat{y} = X\hat{\theta}, \tag{9}$$

where $\hat{y}$ is the estimate of $y$, $\hat{\theta}$ is the estimate of the unknown parameters, and $X$ is a known matrix containing the measured system information. Defining the estimated error as

$$e(k) = y(k) - \hat{y}(k) = y(k) - X\hat{\theta}, \tag{10}$$

and specifying the loss function for this estimate as

$$I(\hat{\theta}) = \frac{1}{2} e^{\mathrm{T}} e, \tag{11}$$

the LS problem becomes one of minimizing this loss function. For the optimal solution, the gradient of $I(\hat{\theta})$ with respect to the parameter $\hat{\theta}$ is zero, i.e.

$$\frac{\partial I(\hat{\theta})}{\partial \hat{\theta}} = -X^{\mathrm{T}} e = -X^{\mathrm{T}}(y - X\hat{\theta}) = 0. \tag{12}$$

Therefore, it follows that

$$\hat{\theta} = (X^{\mathrm{T}}X)^{-1}X^{\mathrm{T}}y. \tag{13}$$

In the current study, the LS method is applied to estimate two matrices, namely the unbalance calculation matrix and the milling vector calculation matrix. In general, if there is a large amount of experimental data, computing the matrix inversion $X^{\mathrm{T}}X$ in Eq. (13) is highly complex and it is therefore impractical to apply the standard LS method directly. By contrast, the adaptive parameter estimation method provides a more feasible approach since it allows the parameters to be continuously adjusted on-line in accordance with new experimental data received [10].

The adaptive parameter estimation technique can be briefly summarized as follows. When new experimental data become available, the current predicted value of $X(k)\hat{\theta}(k-1)$ is compared with the actual output value, $y(k)$, to give an estimated error, i.e.

$$e(k) = y(k) - X^{\mathrm{T}}(k)\hat{\theta}(k-1). \tag{14}$$

The new parameter estimate is then calculated as

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \gamma(k)e(k) \tag{15}$$

with

$$\gamma(k) = \frac{P(k-1)X(k)}{X(k)^{\mathrm{T}}P(k-1)X(k) + \lambda} \quad \text{and} \quad P(k) = \frac{1}{\lambda}(I - \gamma(k)X^{\mathrm{T}}(k))P(k-1). \tag{16}$$

In Eq. (16), the initial value of matrix $P$ is specified as $P(0) = \alpha I$, in which $\alpha$ is a constant and $I$ is an identity matrix. Starting with an initial estimate, $\hat{\theta}(0)$, and the corresponding matrix $P(0)$, $\hat{\theta}$ can be sequentially updated as new experimental data are acquired. This parameter estimation algorithm has the advantage that it computes the parameters step-by-step without repeatedly calculating the matrix solution as in the standard LS method. Furthermore, the algorithm allows for the straightforward update of $\hat{\theta}$ as the number of measurements increases, and therefore permits the slowly varying parameters of the system to be tracked.

### 2.4. Proposed balancing scheme

Fig. 2 presents the overall scheme of the proposed adaptive balancing system comprising the unbalance calculation module, the unbalance correction module, and the parameter estimation module. Before starting the balancing procedure, an off-line process using the LS method is performed to determine the unbalance calculation matrix ($\theta_{YU}$) and the initial milling vector calculation matrix ($\theta_{YX0}$). Subsequently, when a start command is received, indicating the need to balance an armature, the unbalance calculation module is triggered to detect and calculate the initial vibration $Y_p(k)$ (including its amplitude and phase) and the initial unbalance of the armature. The required material removal milling vector, $X(k)$, is then determined by the unbalance correction module using matrix $\theta_{YX0}$, i.e.

$$X(k) = \theta_{YX0} Y_p(k). \tag{17}$$

With Eq. (17), the cutting depth and positions required for unbalance correction are presented in $X(k)$. The armature is then milled in accordance with the information specified in the vector $X(k)$. After milling, the armature is retested by the unbalance calculation module to determine the residual vibration, $Y_r(k)$, and the residual unbalance, $U_r(k)$. If the residual unbalance amount satisfies the specified ISO grade, the balancing process is complete. Otherwise, the armature must be milled again, i.e., further slots must be milled in the armature. When the armature has been successfully balanced, the material removal milling vector $X(k)$, the

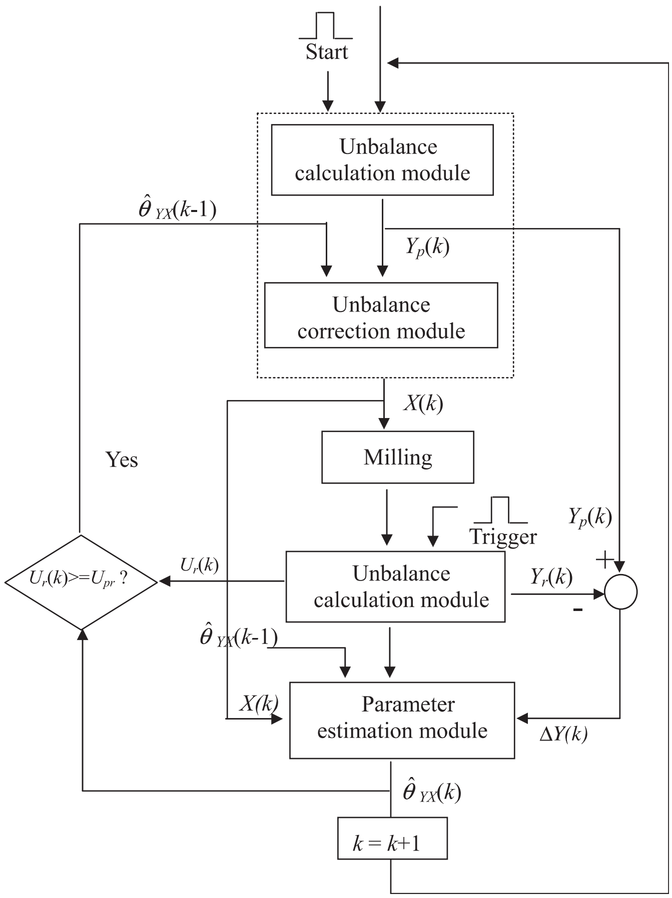


Fig. 2. Adaptive balancing system with unbalance calculation module, unbalance correction module, and parameter estimation module.

previous estimated influence coefficient matrix $\hat{\theta}_{YX}(k-1)$, and the difference between the initial vibration and the residual vibration ($\Delta Y(k) = Y_p(k) - Y_r(k)$) are fed to the parameter estimation module to estimate the new influence coefficient matrix $\hat{\theta}_{YX}(k)$ in accordance with the error equation as following equation:

$$e(k) = \Delta Y(k) - \hat{\theta}_{YX}^{-1}(k-1)X(k) \tag{18}$$

and Eqs. (15) and (16). This calculation is performed after each balancing run and the parameters of the influence matrix are updated accordingly.

In the balancing process, the value of the residual unbalance, $U_r(k)$, is continuously monitored by the system. If $U_r(k)$ is found to increase monotonously, indicating that the cutter or some of the mechanical components of the milling machine are becoming worn, the original influence coefficient matrix, $\theta_{YX0}$, is replaced by the most recent estimated matrix such that the balancing system automatically compensates for changes in the wear condition of the milling machine.

## 3. Experiments

### 3.1. Experimental set-up

As shown in Fig. 3, the hardware components of the unbalance measurement machine include a DC driving motor, an infrared-ray sensor, two vibration sensors, and a mechanism for supporting the armature. In the balancing process, the armature is driven by the driving motor at a speed of 1600 rev/min (27 Hz) and its speed is regulated using a three-phase incremental encoder ($A$, $B$, $Z$ phase) attached to the driving motor. The rotor support mechanism is a soft type with a natural frequency of 8 Hz. The vibrations of the armature are detected via two vibration sensors located on either side of the armature support mechanism. The sampling time of the controlling system is specified as 0.0002 s. Since the oscillatory amplitudes of the rotor are very small and are contaminated by noise, an analog circuit is constructed to filter and amplify the vibration signals. The phase angle of the unbalance is measured by using an infrared-ray sensor (CNY70) to detect the appearance of a mark of reference point placed on the rotor.

The unbalances are corrected by removing an appropriate amount of material from prescribed locations on the rotor. In this study, the material removal method is implemented using the milling machine shown in Fig. 3. The milling apparatus includes a pneumatic driven armature fixture, a cutter driven by an AC motor,

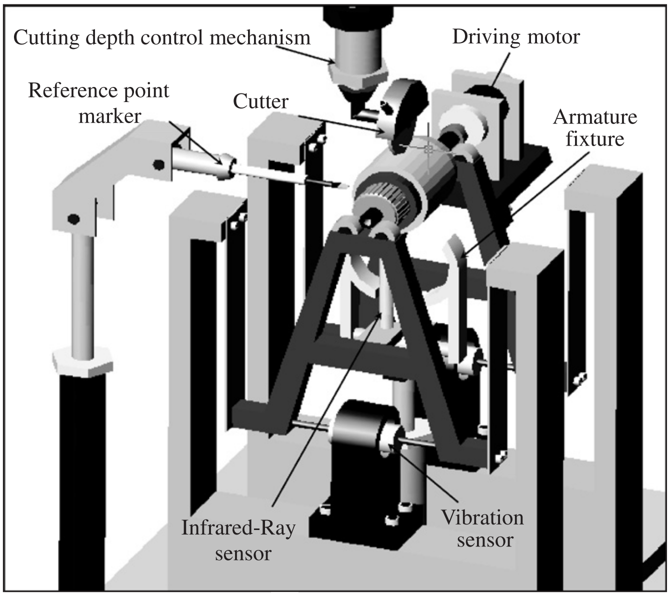


Fig. 3. Configuration of hardware components in unbalance measurement and milling machine.

and a cutting depth control mechanism. The fixture is used to fasten the armature during the milling process. The infrared-ray sensor and the armature driving motor are used to control the cutting position and to avoid damaging the armature coil during machining. The cutting depth control mechanism regulates the cutting depth via a servomechanism. As described in Section 2.4, in the proposed balancing scheme, the unbalance amount is converted to a milling vector in which the cutting locations and depths are specified. In accordance with the information contained in this vector, a U-shaped cutter is employed to cut several slots in the center of the poles on the laminations of the armature.

The real-time control ability of the Matlab XPC system is exploited to construct a system controller with which to regulate the armature speed, calculate the unbalance amount, perform the parameter estimation, and control the cutting operation. The XPC system consists of a host PC and a client PC. The host PC provides the platform for the editing of controlling program written in Simulink while the client PC executes the controlling program in the DOS environment. The two PCs communicate with one another via RS232 or TCP/IP.

### 3.2. Experiments

The armature used in the current balancing experiments is taken from a commercial automobile starting motor (UNIPOINT) and has a weight of 800 g, a length of 116 mm and a diameter of 55 mm, as shown in Fig. 1. For such armatures, ISO 1940 generally specifies a balance grade of G6.3. This particular specification states that at a balancing speed of 1600 rev/min, the maximum permissible residual unbalance amount is 30 g mm. Since balancing is performed dynamically on two planes of the armature, the maximum permissible residual unbalance amount on each plane is therefore 15 g mm. Similarly, for the higher grade of G2.5 (taken in this study as a target when evaluating the performance of the current balancing scheme), the maximum permissible residual unbalance amount on each plane is 6 g mm.

#### 3.2.1. Evaluating precision of unbalance measurement system

The experiments commenced by evaluating the precision of the unbalance measurement data obtained by the proposed system. A total of 10 tests were performed in which various masses were added to 10 different armatures and the corresponding vibratory responses recorded. Applying the LS method to the experimental data, the following unbalance calculation matrix was obtained:

$$\theta_{YU} = \begin{bmatrix} 6.59\angle 167^\circ & 1.47\angle 159^\circ \\ 0.62\angle -6^\circ & 8.32\angle 166^\circ \end{bmatrix}. \tag{19}$$

Theoretically, two added masses and three vibration measurements on the same armature are sufficient to determine this matrix. However, because the experimental vibration signals are inevitably contaminated by a certain level of noise, a larger volume of test data is desirable in order to guarantee a more unbiased and consistent estimate of the matrix. As described previously, the matrix in Eq. (19) describes the relationship between the armature unbalance and the corresponding vibratory response. The location and amount of the armature unbalances corresponding to the vibration measurements $Y_A$ and $Y_B$ can therefore be determined from

$$\begin{bmatrix} U_A \\ U_B \end{bmatrix} = \theta_{YU}^{-1} \begin{bmatrix} Y_A \\ Y_B \end{bmatrix}. \tag{20}$$

In the proposed balancing system, the matrix $\theta_{YU}$ is used to evaluate the initial and residual unbalances. To test the repeatability of the proposed system, the unbalance of an armature with an unknown unbalance was measured 10 times. Table 1 presents the measurement results obtained for the magnitudes and phase angles of the left-hand and right-hand side unbalances in each of the ten trials. The means and variations of the respective quantities are shown at the foot of the table. It is observed that the variances of the measured unbalance characteristics and the balancing speed, respectively, are very small. Therefore, it can be inferred that the repeatability of the unbalance calculation and the speed control of the proposed system are satisfactory.

Table 1
Repeatability of balancing system in measuring armature unbalance

| Test number | Rotation speed (Hz) | Unbalance on right-hand side | | Unbalance on right-hand side | |
|---|---|---|---|---|---|
| | | Magnitude (g mm) | Phase angle (°) | Magnitude (g mm) | Phase angle (°) |
| 1 | 27.78 | 3.86 | 161 | 6.55 | 277 |
| 2 | 27.62 | 3.97 | 157 | 6.45 | 276 |
| 3 | 27.59 | 3.98 | 159 | 6.6 | 275 |
| 4 | 27.86 | 4 | 155 | 6.7 | 277 |
| 5 | 27.78 | 4 | 153 | 7 | 275 |
| 6 | 27.60 | 3.8 | 161 | 6.7 | 275 |
| 7 | 27.50 | 4.1 | 154 | 6.47 | 274 |
| 8 | 27.70 | 3.8 | 159 | 6.5 | 276 |
| 9 | 27.60 | 4 | 161 | 6.5 | 274 |
| 10 | 27.70 | 4 | 154 | 6.7 | 272 |
| Mean value | 27.673 | 3.951 | 157.4 | 6.587 | 275.1 |
| Variance | 0.0122 | 0.0097 | 10.267 | 0.031 | 2.32 |

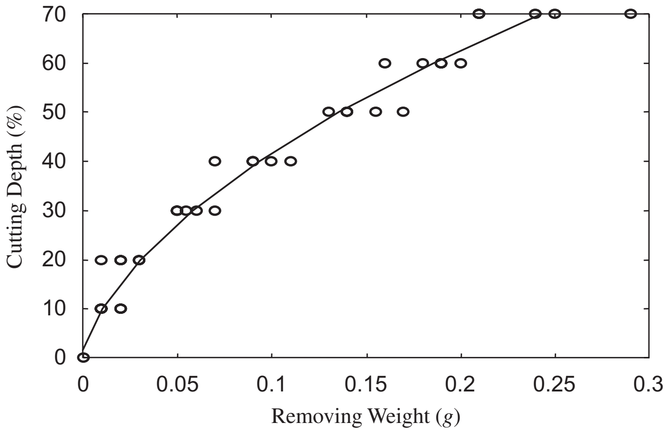


Fig. 4. Relationship between cutting depth (in percentage of the maximum depth) and material removal weight. (*Note*: "o" indicates experimental data, while solid line indicates the curve fitting function.)

### 3.2.2. Unbalance correction using conventional method

Balancing a motor armature using the conventional material removal method involves the following steps: (1) determine the influence coefficient matrix of the armature, $\theta$; (2) calculate the required correction mass vector, $U_c$ in Eq. (8); and (3) transform the correction mass vector to a material removal milling vector, $X = [X_A \; X_B]^T$.

As shown in Fig. 4, in the current experimental system, the removal weight ($w_r$) is a nonlinear function of the cutting depth ($x$). Applying the curve fitting technique, it is found that the two variables are related by an expression of the form

$$w_r = -0.0019 + 0.089x + 0.37x^2, \tag{21}$$

with this function, the milling vector $X$ corresponding to a required correction mass vector $U_c$ can be determined accordingly.

In most practical cases, the unbalance amount of the armature is quite large. Since the maximum removal weight of the cutter is limited, this implies that many individual cuts are required for any given milling vector $X$ (see Fig. 5). Denoting the maximum allowable cutting depth of the cutter as $x_m$, a milling vector, $X$, with

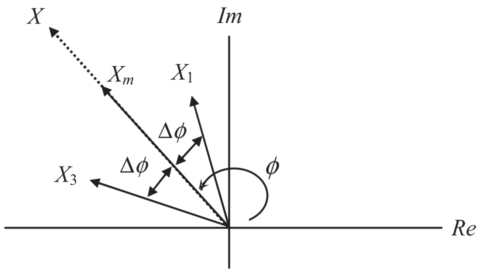


Fig. 5. Decomposition of a milling vector.

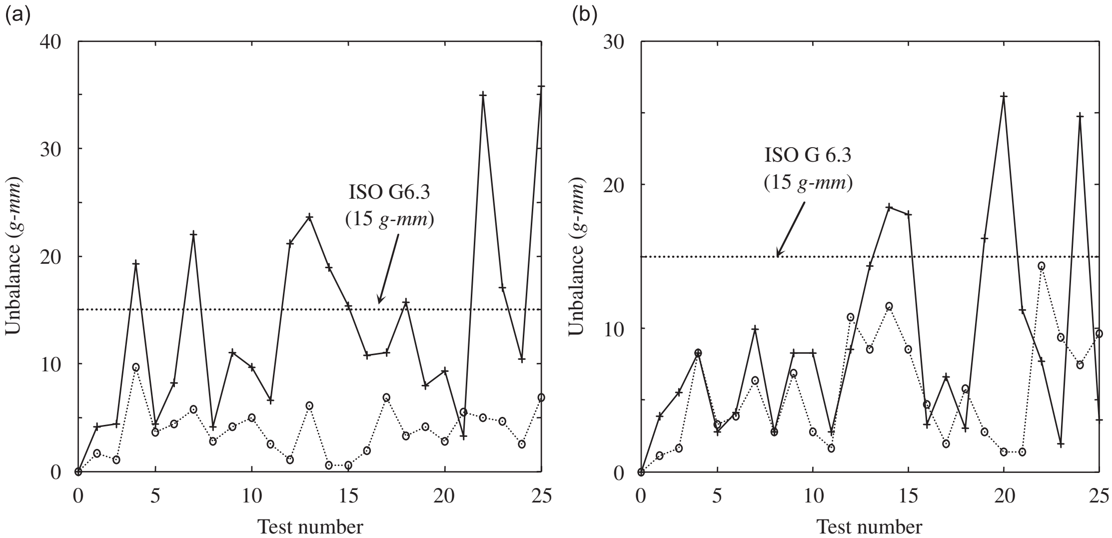


Fig. 6. Balancing results of Case I using influence coefficient matrix $\theta_{YU}$: (a) unbalances on left-hand side of armature; and (b) unbalances on right-hand side. (*Note*: data lines marked with "+" indicate initial unbalances, while those marked with "o" indicate residual unbalances.)

a magnitude of $x$ and a phase angle of $\phi$ can be decomposed as

$$X = X_m + X_1 + X_3 = x_m\angle\phi + x_1\angle(\phi - \Delta\phi) + x_1\angle(\phi + \Delta\phi). \quad (22)$$

Note that Eq. (22) is intended here for illustration purposes only. In real applications, the number of cuts required for a given milling vector depends on its magnitude.

Using the matrix $\theta_{YU}$ in conjunction with Eqs. (21) and (22), a series of balancing experiments was performed using an initial set of 25 armatures. Note that for convenience, these experiments are referred to collectively as Case I in the following. The corresponding balancing results are presented in Fig. 6, in which the left panel presents the unbalances (initial and residual) on the left-hand side of the armature, while the right panel presents those on the right-hand side. The balance tolerance prescribed by the ISO 1940 G6.3 specification is superimposed on both figures for reference purposes. It is observed that the residual unbalances of the balanced armatures all satisfy the ISO G6.3 grade regardless of the amount of the initial unbalance. However, the balanced armatures fail to satisfy the more stringent requirements of the ISO G2.5 specification. This may well be a result of too much approximation in the matrix transformations from the vibration measurements to the amount of unbalance and subsequently to the milling vector.

### 3.2.3. Unbalance correction using proposed method

In this section, the influence coefficient matrix, $\theta_{YX}$, is used to relate the vibration measurement directly to the milling vector, i.e. $X(k) = \theta_{YX}\, Y(k)$. The purpose of this approach is to reduce the errors resulting from the

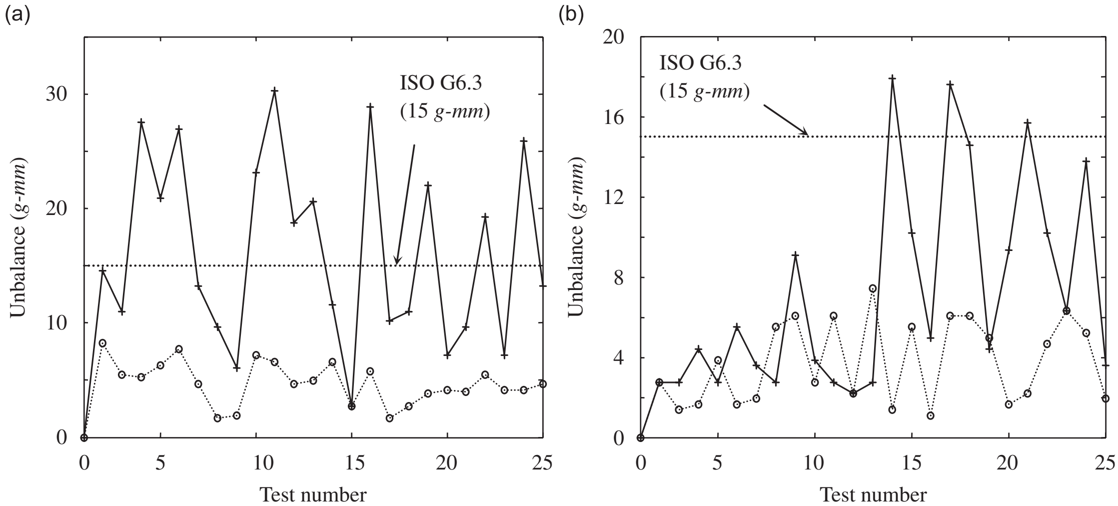


Fig. 7. Balancing results of Case II using influence coefficient matrix $\theta_{YX1}$: (a) unbalances on left-hand side of armature; and (b) unbalances on right-hand side. (*Note*: data lines marked with "+" indicate initial unbalances, while those marked with "o" are residual unbalances.)

various matrix transformations between these two vectors. To obtain the matrix ($\theta_{YX}$), 5 cuts of various depth were made on an armature in different positions on the two balancing planes. For each cut, the difference between the vibration signal before and after the cut was recorded. Applying the LS method to the experimental vibration data, the following matrix was obtained:

$$X(k) = \theta_{YX1}\Delta Y(k), \quad \theta_{YX1} = \begin{bmatrix} 0.28\angle 159^\circ & 0.1\angle 138^\circ \\ 0.175\angle 168^\circ & 0.39\angle 170^\circ \end{bmatrix}. \tag{23}$$

Using this matrix, a series of balancing experiments was carried out using a second group of 25 armatures. The balancing results obtained from these experiments (referred to henceforth as Case II) are presented in Fig. 7. Comparing Figs. 6 and 7, it is apparent that the variances of the residual unbalances in Case II (i.e. 4.1 and 4.6 for the left- and right-hand side unbalances, respectively) are significantly lower than those of Case I (5.5 and 15.1 for the left- and right-hand side unbalances, respectively). Although using the influence coefficient matrix to relate the vibration measurements directly to the milling vector results in an improved balancing performance, the balancing scheme is still unable to satisfy the ISO G2.5 grade. This may well be because the influence coefficient matrix used in Case II was obtained using just five sets of vibration data, with the result that the calculated influence coefficient matrix fails to fully characterize the armature and the balancing system. Although this problem can be resolved by acquiring a greater volume of vibration data, this is impractical in the field because the exact number of measurements required is unknown. Moreover, the data acquisition process is time-consuming.

Accordingly, the experimental investigation next considered the case where the adaptive parameter estimation technique was employed to specify appropriate values of the influence coefficient matrix parameters. Matrix $\theta_{YX1}$ in Eq. (23) was utilized as an initial estimate, and an off-line process was performed to determine a new influence coefficient matrix recursively using Eqs. (14)–(16). The balancing data obtained from Case II, including the initial unbalances, the milling vectors, and the residual unbalances, were used as the input/output data of the parameter estimation algorithm. In the calculations, at each balancing run, the new experimental accompanied with the previous data were randomly picked and then supplied to the parameter estimation algorithm repetitively until the parameters in the influence coefficient matrix converged. The parameters $\lambda$ and $P(0)$ ($= \alpha I$) in Eq. (16) determine the nature of the estimation process. For example, a larger value of $\alpha$ (and hence a larger $P(0)$) accelerates the rate of convergence of the estimation process, but does so at the cost of larger transients. Conversely, a smaller value of $\lambda$ can highlight the importance of new

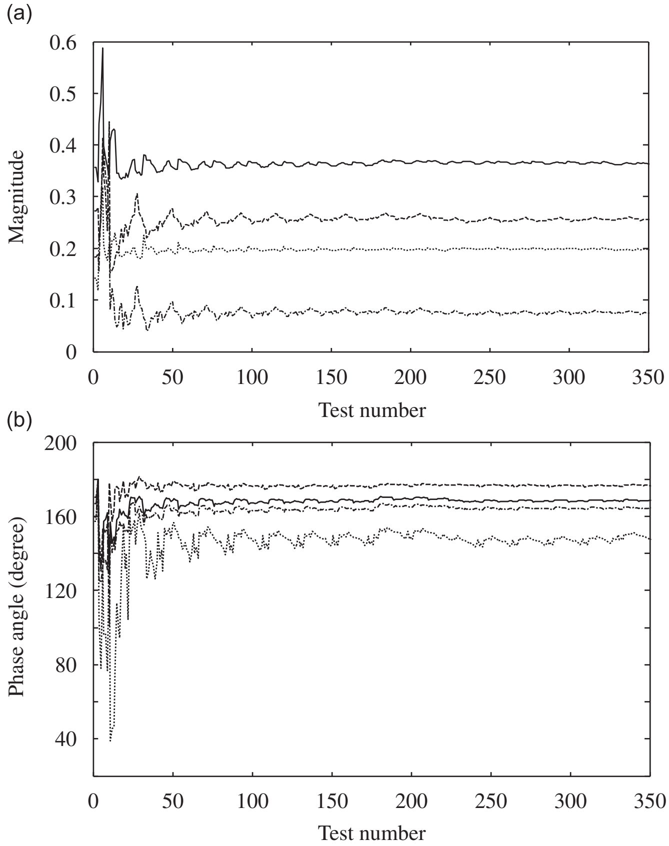


Fig. 8. Parametric convergent trajectories of influence coefficient matrix ($\theta_{YX2}$) during estimation: (a) magnitudes; and (b) phase angles.

coming data (allows the more recent data to receive a greater significance in the estimation process), but may cause the estimated parameters to vary drastically. Fig. 8 shows the parametric convergent trajectories of the estimated influence coefficient matrix parameters for $\lambda = 0.999$ and $\alpha = 1000$. It can be seen that all of the parameters converge successfully to their final values. Expressing the converged parameter values in matrix form, the following influence coefficient matrix is obtained:

$$\theta_{YX2} = \begin{bmatrix} 0.26\angle 176.4^\circ & 0.08\angle 149^\circ \\ 0.197\angle 165^\circ & 0.364\angle 169.3^\circ \end{bmatrix}. \tag{24}$$

Comparing the parameters in $\theta_{YX2}$ with those in $\theta_{YX1}$ (see Eq. (23)), it is observed that the magnitudes of the entries in $\theta_{YX2}$ are slightly lower than those in $\theta_{YX1}$. Physically, this implies that the amount of material removed for a given initial unbalance is lower when matrix $\theta_{YX2}$ is applied, that is, the chance of over-cutting can be reduced. Consequently, the residual unbalance will be decreased.

In the third series of experimental trials (designated as Case III), the balancing scheme shown in Fig. 2 was used to balance a third group of 25 field armatures using the influence coefficient matrix $\theta_0 = \theta_{YX2}$ (see Eq. (24)). The aim of the balancing operation was to reduce the residual unbalance of the armatures to such an extent that the armatures satisfied a higher grade of ISO G2.5. Parameter estimation was carried out for each balancing run and the parameters in the influence coefficient matrix were updated accordingly. The balancing results are presented in Fig. 9. Note that till Sample 16 balancing run, the newly estimated influence coefficient matrices were applied to calculate the milling vectors, since the value of the monitored residual unbalances between Samples 13 and 15 runs showed to increase monotonously beyond the limit of the ISO G2.5 grade. It was inferred that the cutter and the system components were becoming worn. Furthermore, during the balancing process, when the residual unbalance of a particular balancing run exceeded the ISO

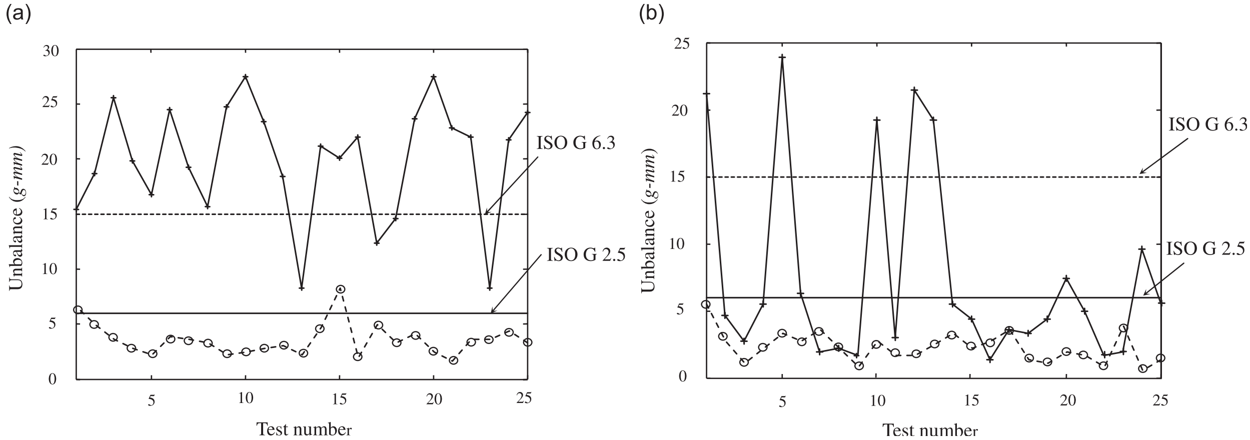


Fig. 9. Balancing results of Case III using influence coefficient matrix $\theta_{YX2}$: (a) unbalances on left-hand side of armature; and (b) unbalances on right-hand side. (*Note*: data lines indicated by "+" indicate initial unbalances, while those marked with "o" indicate residual unbalances).

Table 2
Comparison of statistical properties of residual unbalances obtained using balancing schemes with different influence coefficient matrices

| Cases | The influence coefficient matrix used in the balancing experiment | Mean value (g mm) | | Variance | |
|---|---|---|---|---|---|
| | | Right-hand side | Left-hand side | Right-hand side | Left-hand side |
| Case I | $\theta_{YU}$ | 3.685 | 5.456 | 5.5 | 15.1 |
| Case II | $\theta_{YX1}$ | 4.582 | 3.61 | 4.1 | 4.6 |
| Case III | $\theta_{YX2}$ | 3.586 | 2.228 | 2.23 | 1.34 |

G2.5 grade limit, the corresponding experimental data was discarded and was not used in the parameter estimation process. The intention here was to prevent the occurrence of sudden changes in the parameter values caused by factors such as human error and other unavoidable external disturbances.

From the results presented in Table 2, which summarizes the statistical properties of the residual unbalances obtained using the different influence coefficient matrices, it is clear that both the variance and the absolute amount of the residual unbalance are considerably lower in Case III than in either Case I or Case II. Furthermore, other than two armatures (Samples 1 and 15) with high unbalance amounts on the left-hand side plane, the remaining balanced armatures satisfy the requirements of ISO G2.5 (6 g mm). Overall, the proposed balancing scheme achieves a 92% success rate in satisfying the ISO G2.5 grade.

In conclusion, the balancing results obtained for Case III reveal that the proposed balancing scheme with the adaptive parameter estimation algorithm provides a better armature balancing performance than the conventional method. Since the relationship between the cutting depth of the milling machine and the removal weight of the armature depends on the degree of wear of the milling cutter and the milling machine components, the parameters in the influence coefficient matrix should be progressively updated as the number of milling operations increases. The Case III experimental results show that the proposed method fulfills this requirement without the need for manual intervention. In other words, the proposed dynamic balancing system provides the potential to realize a fully automated rotor balancing operation.

## 4. Conclusions

This study has developed a dynamic balancing system for motor armatures in which the influence coefficient matrix is calculated using an adaptive parameter estimation method. The results have shown that the balancing quality is better than that obtained using the conventional influence coefficient method.

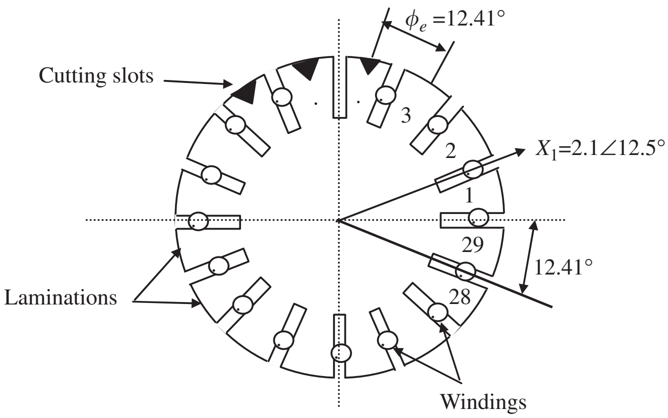


Fig. 10. Simplified diagram of cross-section of the experimental motor armature.

In general, errors in the milling position may potentially result in a significant amount of residual unbalance. The main sources of the milling position error include round-off and accumulating errors in the various calculations. Due to constrain of the armature's structure, the cutting center must coincide with the center of the poles on the laminations of the armature in order to avoid destroying the armature windings. As a result, even a small angular calculating or round-off error will at least result in a milling position error of $\phi_e = 360^\circ/29 = 12.41^\circ$ for the current experimental armature with 29 poles (as illustrated in Fig. 10). According to our estimation result based on the present experimental data, this milling angular error can cause a residual unbalance of approximately 8–66% of the initial unbalance. In the most extreme case, when the milling vector is calculated to be $X_1 = 2.1\angle 12.5^\circ$ (as shown in Fig. 10), the question arises as to whether the cutting center should be placed at Pole 1 or at Pole 2. Since all the vibration measurements used for the calculations are contaminated by certain levels of noises having its own distribution, it is difficult to construct reliable guidelines for this decision-making process. Accordingly, a future study will consider the application of fuzzy-logic theory to resolve this problem. In the anticipated approach, the vibration measurements will be defined as fuzzy variables and a fuzzy relationship between the vibration measurements and the milling vector will be established. Finally, the milling vector for a given vibration measurement will be determined by a process of fuzzy inference.

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